

MODULE 10: APPLYING CONSTANT RATES TO MEASUREMENT

In measuring lengths, most of us know that there are 12 inches per foot. Perhaps not so many of us know that there are 110 fathoms per furlong. Nevertheless, we should not confuse an *unfamiliar* rate with a *difficult* problem. We handle constant rates in the same way, whether the rates are familiar or unfamiliar.

Example 1

If there are 110 fathoms per furlong, how many fathoms are there in 7 furlongs?

Since there are 110 fathoms in each furlong, there will be 110 fathoms, 7 times, or:

7×110 fathoms
in 7 furlongs.

$7 \times 110 = 770$. Therefore there are
770 fathoms in 7 furlongs.

Admittedly, it might seem more helpful if we knew that a fathom was 6 feet and that a furlong was $\frac{1}{8}$ of a mile. However, once we know the relationship between fathoms and furlongs, we can convert from one to the other without having to know any other information.

Example 2

How many furlongs are there in 550 fathoms?

Every furlong is 110 fathoms. Since 550 is the 5th multiple of 110, there are 5 furlongs in 550 fathoms.

Answer: 770

Using common fraction notation:

$$\frac{7 \text{ furlongs}}{1} \times \frac{110 \text{ fathoms}}{1 \text{ furlong}} =$$

$$\frac{7 \text{ furlongs} \times 110 \text{ fathoms}}{1 \times 1 \text{ furlong}} =$$

$$\frac{7 \times 110}{1} \text{ fathoms} =$$

770 fathoms

Answer: 5

We could "count":

1 furlong = 110 fathoms

2 furlongs = 220 fathoms

3 furlongs = 330 fathoms

4 furlongs = 440 fathoms

5 furlongs = 550 fathoms

Examples 1 and 2 are both illustrations of the formula:

$$\text{Number of furlongs} \times 110 = \text{Number of fathoms}$$

In Example 1, 7 modified the number of furlongs and this leads to:

$$7 \times 110 = \text{Number of Fathoms}$$

In Example 2, 550 modified the number of fathoms and this leads to:

$$\text{Number of Furlongs} \times 110 = 550$$

which is a division problem whose solution is:

$$\text{Number of Furlongs} = 550 \div 110 = 5$$

Sometimes a rate is so common that it is assumed that we know it.

Example 3

How many feet are there in 6 yards?

If we knew the number of feet per yard, we'd solve this problem in the same way as we would solve any rate problem. Namely:

$$\frac{6 \text{ yards}}{1} \times \frac{? \text{ feet}}{1 \text{ yard}} =$$

$$\frac{6 \text{ yards} \times ? \text{ feet}}{1 \times 1 \text{ yard}} =$$

$$\frac{6 \times ?}{1} \text{ feet} =$$

$$(6 \times ?) \text{ feet}$$

If we know that there are 3 feet in a yard, we may replace ? by 3 to get

$$(6 \times 3) \text{ feet or } 18 \text{ feet.}$$

If we didn't know the number of feet in one yard, we could look it up. For your convenience, here is

That is:

$$\text{furlongs} \times \frac{110 \text{ fathoms}}{1 \text{ furlong}} = \text{fathoms}$$

In terms of common fractions:

$$\frac{550 \text{ fathoms}}{1} \times \frac{1 \text{ furlong}}{110 \text{ fathoms}} =$$

$$\frac{550}{110} \text{ furlongs}$$

Answer: 18

We use the question mark (?) to remind us that we have not yet been told the number of feet in a yard.

This makes sense. Namely, no matter how many feet there are in 1 yard, there are 6 times that number in 6 yards. That's what constant rate really means.

a chart of the most commonly used rates for lengths in the English system.

Lengths in the English System

The Commonly-Used Rates

12 inches = 1 foot

3 feet = 1 yard

1,760 yards = 1 mile

A Few Not-So-Common Rates

1 fathom = 6 feet

1 furlong = $\frac{1}{8}$ of a mile

1 league = 3 miles

From this table of rates, we can deduce other rates which aren't directly in our table.

Example 4

How many feet are there per furlong?

Method 1

A furlong is $\frac{1}{8}$ of a mile. Since a mile is 1,760 yards, a furlong is also $\frac{1}{8}$ of 1,760 yards; or 220 yards.

Then since each yard is 3 feet, 220 yards will contain 3 feet 220 times. That is, there are 3×220 , or 660, feet in a furlong.

Method 2 (Using the Common Fraction Notation)

The problem can be worded as:

1 furlong = _____ feet

From the table we can convert furlongs to miles:

$$\frac{1 \text{ furlong}}{1} \times \frac{1 \text{ mile}}{8 \text{ furlongs}} = \frac{1}{8} \text{ miles}$$

The English system refers to the measurement system used primarily in America. About 95% of the world uses the Metric System, which we'll discuss later in this module.

There are other English measurements that aren't mentioned here. Should you encounter one, you can look it up in a dictionary to see how it is related to a more familiar measurement.

Answer: 660

Get the idea? feet per furlong isn't in our chart. But feet per yard, miles per furlong, and yards per mile are in our chart.

If you don't like fractions, notice that $\frac{1}{8}$ mile per furlong means:

$$\frac{1 \text{ mile}}{8 \text{ furlongs}} \text{ or } \frac{8 \text{ furlongs}}{1 \text{ mile}}$$

and this is the same as saying that there are 8 furlongs per mile.

If this rate were in our table, we'd have no need to go any further.

$\frac{1}{8}$ of a mile per furlong is the same as saying that there are 8 furlongs per mile.

We can then convert $\frac{1}{8}$ miles to yards:

$$\frac{1 \text{ mile}}{8} \times \frac{1,760 \text{ yards}}{1 \text{ mile}} =$$

$$\frac{1,760}{8} \text{ yards} =$$

220 yards

Finally we can convert 220 yards to feet:

$$\frac{220 \text{ yards}}{1} \times \frac{3 \text{ feet}}{1 \text{ yard}} =$$

660 feet

Putting all of these steps into one line

we have:

$$\frac{1 \text{ furlong}}{1} \times \frac{1 \text{ mile}}{8 \text{ furlongs}} \times \frac{1,760 \text{ yards}}{1 \text{ mile}} \times \frac{3 \text{ feet}}{1 \text{ yard}} =$$

$$\frac{1 \times 1 \times 1,760 \times 3}{1 \times 8 \times 1 \times 1} \text{ feet} =$$

220

$$\frac{1 \times 1 \times 2,760 \times 3}{1 \times 8 \times 1 \times 1} \text{ feet} =$$

1

$$\frac{660}{1} \text{ feet or } 660 \text{ feet}$$

The English system was invented before place value had gained any wide acceptance. For this reason, people wanted rates that translated easily into fractional parts. For example, a foot was divided in to twelfths because it was easy to take fractional parts of 12 and still deal with whole numbers. Even when we say a half-dozen, we think of the whole number 6. 60 was used as the number of minutes in an hour because in this way $\frac{1}{2}$ -hour is 30 minutes; $\frac{1}{3}$ hour is 20 minutes; $\frac{1}{4}$ hour is 15 minutes; $\frac{1}{5}$ hour is 12 minutes; and $\frac{1}{6}$ of any hour is 10 minutes. Yet when we say 10 minutes, or 12 minutes, or 15 minutes it seems as if we're dealing with whole numbers.

1,760 yards = 1 mile is in our chart.

3 feet = 1 yard is a fact we can read from our table.

The check is that once we've cancelled all common denominations, only the desired denomination is present. The pattern used here is that we first cancelled "furlongs" to get "miles". Then we cancelled "miles" to get "yards". Finally we cancelled "yards" to get "feet". The order was determined by what we were given in the chart. For example, if the chart had told us that there were 660 feet per furlong, we'd have had the answer at once.

Notice the psychological advantage of saying 1 inch (a whole number) instead of $\frac{1}{12}$ of a foot (a fraction). This is how ancient peoples disguised the existence of fractions.

After place-value was invented, it became desirable in many instances to use ten as the basis for all rates. The system that utilizes this approach in modern science is called the Metric System.

Just as the "foot" was taken as the basic unit of length in the English System, a different length called a meter was taken as the basic unit of length in the metric system.

Instead of dividing a meter into twelfths, it was divided into tenths; and each of these parts was called a decimeter.

Note

As a prefix "deci" always means "one-tenth of". Thus, "decimeter" means "one-tenth of a meter". It may be easier to remember if you recall that "dime" is a contraction of "decimal". That is we may think of a dime as being an abbreviation for "decidollar" which means "one-tenth of a dollar"

Example 5

How many decimeters are there in 1 meter?

We already know from the definition that a decimeter is one-tenth of a meter.

That is:

$$1 \text{ decimeter} = \frac{1}{10} \text{ meters}$$

$$10 \times 1 \text{ decimeter} = 10 \times \frac{1}{10} \text{ meters, or}$$

$$10 \text{ decimeters} = \frac{10}{10} \text{ or } 1 \text{ meter}$$

Our money system is an easy example of this. For example there are 10 cents per dime and 10 dimes per dollar. At one time there was a coin called a mill that was $\frac{1}{10}$ of a cent. That is, there were 10 mills per cent.

The length "foot" is derived from the fact that it was the actual length of a king's foot. A "meter" is based on a more scientific measurement that is of no consequence to us here.

"deci" is a Latin prefix indicating "ten" or "tenth" Think for example of the words "decade" and "decimal"

There is no such word as "decidollar". We're simply showing how it could have been used in place of "dime"

Answer: 10

If the vocabulary bothers you replace "meter" by "dollar". There are 10 dimes per dollar. Hence there are 10 decidollars per dollar.

In this same vein, there are 10 decimeters per meter.

Here's another way to see this:

$$1 \text{ decimeter per } \frac{1}{10} \text{ meter}$$

means:

$\frac{1 \text{ meter}}{10 \text{ decimeters}}$ means the same as $\frac{10 \text{ decimeters}}{1 \text{ meter}}$ and in this form it's read as 10 decimeters per meter.

Notice that "deci" will always mean "one-tenth of" even if you are unfamiliar with the denomination being described.

Example 6

How many decigrams are there in 1 gram?

A decigram is one-tenth of a gram. Hence 10 decigrams will be a whole gram.

Example 7

How many deciamperes are there in 1 ampere?

Again, a deciampere is one-tenth of an ampere. Hence there will be ten deciamperes in each ampere.

So in terms of adjectives and nouns, "deci" as an adjective is always "one-tenth of". "one-tenth of" what depends on the noun that "deci" modifies.

The metric system then continues the same way as decimals did when we used place value. After we subdivide a meter into tenths, we then subdivide each of the tenths into tenths. A tenth of a decimeter is called a centimeter. In other words,

10 centimeters = 1 decimeter

Example 8

How many centimeters are there in a meter?

There are 10 centimeters per decimeter and 10 decimeters per meter. Hence there are 10 X 10 or 100 centimeters per meter.

Answer: 10

A gram is the basic unit of weight in the metric system. But we don't have to know this to do Example 6.

Answer: 10

An ampere is a measure of electrical current, but we don't have to know this to do Example 7.

Again in terms of money, we subdivided a dollar into tenths, called dimes. We then subdivided dimes into tenths, and called each of these parts, cents.

Answer: 100

Again you may prefer to think in terms of money. There are 10 cents per dime but 100 cents per dollar. In this way, you may think of a cent as an abbreviation for a "centidollar"

Other ways of seeing this result include:

$$1 \text{ meter} = 10 \text{ decimeters}$$

$$= 10 \times 1 \text{ decimeter}$$

$$= 10 \times 10 \text{ centimeters}$$

*We replaced 1 decimeter by
10 centimeters*

and in terms of common fractions:

$$\frac{1 \text{ meter}}{1} \times \frac{10 \text{ decimeters}}{1 \text{ meter}} \times \frac{10 \text{ centimeters}}{1 \text{ decimeter}} =$$

*We keep cancelling labels
until we get to the one we
want.*

$$\frac{1 \text{ meters} \times 10 \text{ decimeters} \times 10 \text{ centimeters}}{1 \times 1 \text{ meter} \times 1 \text{ decimeter}} =$$

$$\frac{100}{1} \text{ or } 100 \text{ centimeters}$$

And the process continues..... Once we have decimeters and centimeters we can subdivide each centimeter into 10 equal parts. Since one-tenth of a hundredth is a thousandth, and since the Latin word for "thousand" is "millum", a tenth of a centimeter became known as a millimeter.

*This is easy to remember if
we think of the Roman numeral
M that stands for 1,000*

For some of today's modern measurements, even a millimeter is too large. This we continue by taking a tenth of a millimeter or a hundredth of a millimeter or a thousandth of a millimeter. These rates can be looked up in scientific tables that appear in unabridged dictionaries should you have the need to use them.

*But we always maintain the
place value system of dividing
by ten each time.*

*A thousandth of a millimeter
is called a micromillimeter*

There are also names given to power-of-ten multiples of meters. While names for tenths, hundredths, and thousandths of a meter are derived from Latin prefixes; the names for ten, a hundred, or a thousand meters are derived from Greek prefixes. In particular:

*Up to now we've only considered
fractional parts of
meters.*

10 meters = 1 dekameter

10 dekameters = 1 hectometer

10 hectometers = 1 kilometer

Confusing a dekameter with a decimeter would be like confusing \$10 with a dime. A dime is like a decidollar, but \$10 is like a dekadollar.

We can memorize these names or we can look them up when we need them. A good rule of thumb is that if we have to use the names often enough we'll remember them simply by repeated usage. Once we've learned the above rates we can derive others.

Example 9

How many meters are there in 1 kilometer?

Answer: 1,000

We convert from kilometers to hectometers, from hectometers to dekameters, and from dekameters to meters. In this way we get:

Sometimes the salary for a job is advertised as \$25K. In this context K stands for a "kilodollar" or \$1,000. Thus \$25K means \$25,000

1 kilometer = 10 hectometers

= 10 X 1 hectometer

= 10 X 10 dekameters

= 100 X 1 dekameter

= 100 X 10 meters

= 1,000 meters

Some abbreviations

m stands for meter

dkm stands for dekameter

hm stands for hectometer

km stands for kilometer

$\frac{1 \text{ km}}{1} \times \frac{10 \text{ hm}}{1 \text{ km}} \times \frac{10 \text{ dkm}}{1 \text{ hm}} \times \frac{10 \text{ m}}{1 \text{ dkm}}$

simplifies to $\frac{1,000}{1}$ meters.

"Kilo" is always the prefix for "thousand".

Example 10

How many grams are there in a kilogram?

Answer: 1,000

A kilogram stands for a *thousand* grams.

Remember, you don't have to know what a gram is to do this problem.

Just say "thousand" instead of "kilo".

But don't confuse "kilo" which stands for "thousand" with "milli" which stands for a "thousandth."

Example 11

How many grams are there in a milligram?

Answer: $\frac{1}{1,000}$ of a gram

A milligram is a "thousandth (of)" a gram.

A "nicer" way of saying this is that there are 1,000 milligrams per gram.

By "nicer" we mean without having to use fractional parts

Note the reciprocal relationship of rates. For example when we say 1,000 milligrams (mg) per gram, we can write it as:

$$\frac{1,000 \text{ mg}}{1 \text{ gram}}$$

As long as the denominations are present this is the same as

$$\frac{1 \text{ gram}}{1,000 \text{ mg}}$$

which we can read as $\frac{1}{1,000} \frac{\text{grams}}{\text{mg}}$ or

$\frac{1}{1,000}$ grams per milligram.

Get the point? We're not saying that 1,000 and 1/1000 are the same. We are saying that:

$$\frac{1,000 \text{ mg}}{1 \text{ gram}} \text{ and } \frac{1 \text{ gram}}{1,000 \text{ mg}}$$

are the same rate.

People who know the English system do not have to know the metric system in order to make measurements. Conversely, the people who know the metric system don't have to know the English system in order to make measurements. However if the two peoples wish to interact then each must be able to translate their measurements into the language of the other.

If this paragraph seems too abstract, again think in terms of money. For example Americans can work with dollars and not know what a ruble is. Russians can work with rubles without knowing what a dollar is. But if Russians and Americans wish to barter, then each must be able to convert to the monetary system of the other.

To convert lengths from the metric system into the English system or from the English system into the metric system, we need know just one rate that relates the two systems. The basic rate we'll use in this course is that:

There are 2.54 centimeters per inch

That is: 2.54 cm = 1 in.

The results we'll use in this course are summarized in Table 1:

<u>Table 1</u>	
<u>Metric Relations</u>	<u>English Relations</u>
1 kilometer = 1,000 meters	1 mile = 1,760 yards
1 meter = 1,000 millimeters	1 yard = 3 feet
or	1 foot = 12 inches
100 centimeters	
The "Cross-Over" or "Bridge"	
2.54 centimeters = 1 inch	

Rather than say 1 deka-meter we'll say 10 meters. Centimeters, millimeters, meters, and kilometers are used more often than are decimeters, dekameters and hectometers.

It is important to notice that we could have used many other cross-overs to relate Metric and English measurements. It's just that once we know one, all others can be expressed in terms of that one.

Example 12

How many millimeters are there in 1 inch?

We already know from Table 1 that there are 2.54 centimeters (cm) per inch and that there are 10 millimeters (mm) per centimeter.

Therefore, we have:

$$\begin{aligned}
 1 \text{ inch} &= 2.54 \text{ centimeters} \\
 &= 2.54 \times 1 \text{ centimeter} \\
 &= 2.54 \times 10 \text{ millimeters} \\
 &= 25.4 \text{ millimeters.}
 \end{aligned}$$

In terms of common fractions:

$$\frac{1 \text{ inch}}{1} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} \times \frac{10 \text{ mm}}{1 \text{ cm}} =$$

$$\frac{1 \cancel{\text{ inch}} \times 2.54 \text{ cm} \times 10 \text{ mm}}{1 \times 1 \cancel{\text{ inch}} \times 1 \text{ cm}} =$$

$$25.4 \text{ mm}$$

Answer: 25.4

NOTE:

There are many tables that give the rate 25.4 millimeters per inch. But to emphasize the properties of constant rates, all length problems in this course should use only those rates given in Table 1.

If Table 1 had used as a "cross-over" that there are 25.4 mm per inch, then Example 12 could have proved that there are 2.54 cm per inch.

Once we know how many centimeters there are in an inch we can use reciprocals to find how many inches there are in a centimeter.

Example 13

If there are 2.54 centimeters per inch, how many inches are there per centimeter?

All we did here was repeat the cross-over rate from Table 1. Now use the common fraction form to get:

$$\begin{array}{l} \text{or} \quad \frac{1 \text{ inch}}{2.54 \text{ cm}} \\ \frac{1}{2.54} \quad \frac{\text{inch}}{\text{cm}} \\ \text{or} \quad \frac{100}{254} \quad \frac{\text{inch}}{\text{cm}} \\ \text{or} \quad \frac{50}{127} \text{ inches per centimeter} \end{array}$$

Caution:

While we assumed that there were 2.54 cm per inch, this is also a rounded off answer. If greater accuracy were needed we might have to extend the 2.54 to more decimal place accuracy. However, for the calculations needed in our course, we may assume that 2.54 cm per inch is exact.

There has been a movement in this country to "think metric". Most of us can't even "think English" when it comes to measurement. Our course is not concerned with "thinking metric". However, in terms of our emphasis on mathematics as a language, if you're given a metric measurement, you can use Table 1 to convert into an English measurement. This can be helpful because most Americans are relatively comfortable with the English System.

Answer: $\frac{50}{127}$ or in decimal form, 0.394 to the nearest thousandth of an inch.

See the constant rate form
inches \times 2.54 = cms
means
inches = cm \div 2.54

If you prefer to avoid using common fractions, rec the answer as 50 inches per 127 inches. In decimal form we can divide 50 by 127 and get 0.394 if we round off to the nearest thousandth of an inch.

That is, because of the nature of how a centimeter and an inch are related, the crossover rate of 2.54 is simply a good approximation just as 0.333 is a good approximation for $1/3$ in most cases.

See the language idea? When you're asked a question in a foreign language, you can translate it into your own language; then answer it in your own language; and then you can translate the answer into the foreign language.

Example 14

Which length is longer, 500 centimeters or 17 feet?

To compare two lengths we need a common denomination. So we can either convert 17 feet to centimeters or we can convert 500 centimeters to feet. For practice let's do it both ways:

Method 1: We convert 17 feet to centimeters.

$$\begin{aligned} 17 \text{ feet} &= 17 \times 1 \text{ foot} \\ &= 17 \times 12 \text{ inches} \\ &= 204 \text{ inches} \\ &= 204 \times 1 \text{ inch} \\ &= 204 \times 2.54 \text{ centimeters} \\ &= 518.16 \text{ centimeters} \\ &\quad \text{which is more than 500 cm.} \end{aligned}$$

Method 2: We convert 500 centimeters to feet.

$$\begin{aligned} 500 \text{ centimeters} &= 500 \times 1 \text{ cm} \\ &= 500 \times \frac{50}{127} \text{ inches} \\ &= \frac{25,000}{127} \\ &= \frac{25,000}{127} \times 1 \text{ inch} \\ &= \frac{25,000}{127} \times \frac{1}{12} \text{ feet} \\ &= \frac{25,000}{1,524} \text{ feet} \\ &= 16 \frac{616}{1524} \text{ feet} \\ &\quad \text{which is less than 17 feet} \end{aligned}$$

Answer: 17 feet

For example 9 is more than 1 but 9 inches is less than 1 foot.

If you want to practice with the common fraction notation and cancel common denominations, feel free to use that method here.

We're using Example 13 here

12 inches per foot means $\frac{12 \text{ inches}}{1 \text{ foot}}$ which is the same as the rate $\frac{1 \text{ foot}}{12 \text{ inches}}$ or 1/12 feet per inch.

It doesn't matter which way you prefer to do Example 14. What is important is that if you're comfortable with the English system and have to deal with a length of 500 centimeters in the metric system, you can use the method of

Example 14 to conclude that 500 centimeters is a little less than 17 feet. Similarly, if you feel more comfortable with the metric system and you have to deal with 17 feet, Example 14 lets you conclude that it is a little more than 500 centimeters.

Possibly the most important thing to keep in mind is that you do not have to know the metric system to use Table 1. In fact, it is knowledge of the metric system that allowed us to construct the table. Let's illustrate this by looking at two situations:

Example 15

At \$2.54 per pound, how much would 12 pounds of ground beef cost?

Answer: \$30.48

For each pound of beef we're paying \$2.54.

So for 12 pounds, we have to pay \$2.54 twelve times.

Therefore, 12 pounds will cost us

$$12 \times \$2.54 \text{ or } \$30.48$$

Example 16

If there are 2.54 centimeters per inch, how many centimeters are there in 12 inches?

Answer: 30.48

For each inch, we have 2.54 centimeters.

So for 12 inches, we have 2.54 centimeters, twelve times. Hence:

$$\begin{aligned} 12 \text{ inches} &= 12 \times 1 \text{ inch} \\ &= 12 \times 2.54 \text{ centimeters} \\ &= 30.48 \text{ centimeters.} \end{aligned}$$

To say that you can't do Example 16 because you don't know the metric system makes as much sense as saying that

Since there are 12 inches in a foot, Example 16 also tells us that there are 30.48 centimeters per foot.

you can't do Example 15 because you don't know the "ground beef" system! You may have to know something about butchery and marketing to arrive at why the price per pound is \$2.54; but once you're told the price per pound it is just a matter of simple rates to find the answer to Example 15. In a similar way, once you are told that there are 2.54 centimeters per inch, it is a matter of simple rates to find the number of centimeters in 12 inches.

The main point is that from a mathematical point of view there is no difference between Examples 15 and 16. However because we are more comfortable with dollars and pounds than with inches and centimeters, Example 15 is psychologically easier for most people than Example 16.

Additional problems are left for the Self-Test but for now we'd like to examine a few other measurements that are commonly made within both the English and the metric systems. Table 2 gives us some rates concerning weights.

Table 2

Metric Relations

1 metric ton (mt) = 1,000 kilograms (kg)

1 kilogram = 1,000 grams

1 gram = 1,000 milligrams (mg)

English Relations

1 ton = 2,000 pounds (lb)

1 pound = 16 ounces

Cross-Over Relationship

454 grams = 1 pound

Using Table 2 we can convert weights between the English and the metric systems simply as an application of constant rates. We'll try just a few examples to make sure that you have the correct idea.

Example 17

How many grams are there in 10 pounds?

Answer: 4,540

The cross-over relationship is:

number of pounds X 454 = number of grams.

So we simply replace the number of pounds by 10 to get:

$$10 \times 454 = 4,540 \text{ grams.}$$

In the language of common fractions:

$$\frac{10 \text{ pounds}}{1} \times \frac{454 \text{ grams}}{1 \text{ pound}} =$$

$$\frac{10 \text{ pounds} \times 454 \text{ grams}}{1 \times 1 \text{ pound}} =$$

$$\frac{4,540 \text{ grams}}{1} \text{ or } 4,540 \text{ grams}$$

These are simply more formal ways of saying that since each pound has 454 grams, 10 pounds will have 10 times 454 grams. Hopefully, the formal way will help you make sure you see the correct arithmetic.

Example 18

How many kilograms are there in 10 pounds?

Answer: 4.54

We already know that 10 pounds is 4,540 grams. So all that remains to do is express 4,540 grams in kilograms. Since there are 1,000 grams in each kilogram, we must divide 4,540 grams by 1,000 to get the number of kilograms. That is:

$$\frac{4,540 \text{ grams}}{1} \times \frac{1 \text{ kilogram}}{1,000 \text{ grams}} =$$

$$\frac{4,540}{1,000} \text{ kilograms} = 4.54 \text{ kilograms.}$$

Note that we could have done Example 18 directly from Table 2 without doing Example 17 first.

Namely:

In terms of rounding off, we sometimes write 4.540 to indicate that our measurement is accurate through the thousandths-place

That is:

of kg X 1000 = # of grams

Hence:

of kg = # of grams ÷ 1,000

Note:

Since 10 pounds = 4.54 kgs, a pound is less than a kilo gram. That is, it takes 10 pounds to measure the same weight as 4.54 kilograms. In other words, the smaller the unit of measurement, the more units we need

$$\frac{10 \text{ pounds}}{1} \times \frac{454 \text{ grams}}{1 \text{ pound}} \times \frac{1 \text{ kilogram}}{1,000 \text{ grams}} =$$

$$\frac{10 \text{ pounds} \times 454 \text{ grams} \times 1 \text{ kilogram}}{1 \times 1 \times 1 \text{ pound} \times 1,000 \text{ grams}} =$$

$$\frac{4,540 \text{ kilograms}}{1,000} = 4.54 \text{ kilograms}$$

Example 19

How many pounds are there in 10 kilograms?
Write the answer as a decimal fraction
rounded off to the nearest hundredth of a
pound.

Method 1

From Example 18, we already know that
there are 10 pounds per 4.54 kilograms.

Recalling that "per" can be replaced by

"÷" we have that:

$$10 \text{ pounds per } 4.54 \text{ kilograms} =$$

$$10 \text{ pounds} \div 4.54 \text{ kilograms} =$$

$$(10 \div 4.54) \text{ pounds per kilogram} =$$

$$2.2026... \text{ pounds per kilogram}$$

Method 2

$$\frac{10 \text{ kg}}{1} \times \frac{1,000 \text{ grams}}{1 \text{ kg}} \times \frac{1 \text{ pound}}{454 \text{ grams}} =$$

$$\frac{10 \text{ kg} \times 1,000 \text{ grams} \times 1 \text{ pound}}{1 \times 1 \text{ kg} \times 454 \text{ grams}} =$$

$$\frac{10 \times 1,000 \text{ pounds}}{454} = \frac{10,000}{454} \text{ pounds}$$

If you compare Examples 18 and 19 you can begin to
see the danger of trying to memorize rates rather than
understand them. The two examples look almost identical;
yet in one case we have to multiply by 454 and in
the other case we have to divide by 454.

Answer: 22.03

We usually round this answer
off to 22 which means that
there are about 2.2 pounds
per kilogram (So if you
want to "think metric", a
kilogram is a little more
than 2 pounds). However
for heavier weights it can
be important to use 2.203
pounds per kilogram.

If we prefer not to round
off, write $10 \div 4.54$ as a
common fraction; namely
 $1,000/454 = 500/227$

In summary:

$$\# \text{ of kg} \times 2.203 = \# \text{ of lbs}$$

So we multiply by 2.203 if
we're given the number of
kgs. but we divide by 2.203
if we're given the number
of pounds. That is, we
multiply when we convert
greater units to lesser ones
and divide when we convert
lesser units to greater
ones.

Most people know enough to
use the 454. But they get
confused as to whether to
multiply or divide.

The arithmetic of the metric system has become more important with the advent of calculators. Calculators work with decimals, and metric rates are based on powers of ten. For example suppose that using a calculator you found that a length was 4.56 meters. Then it would be correct to say that the length was 4 meters plus 56 centimeters. In other words, since there are 100 centimeters per meter, 0.56 meters is 56 centimeters.

But how would we convert 4.56 feet to the nearest inch? We can't say that 4.56 feet is 4 feet plus 56 inches! Rather we have to think something like this:

We could have, if there had been 100 inches per foot.

$$\begin{aligned} 0.56 \text{ feet} &= 0.56 \times 1 \text{ foot} \\ &= 0.56 \times 12 \text{ inches} \\ &= 6.72 \text{ inches.} \end{aligned}$$

Since most of us think in the English system and use calculators it might be important to make sure that you can translate decimal fractions correctly.

Example 20

Convert 0.45 hours into minutes.

Answer: 0.45 hr = 27 min

0.45 hours means

$$\begin{aligned} 0.45 \times 1 \text{ hour} &= 0.45 \times 60 \text{ minutes} \\ &= 27 \text{ minutes} \end{aligned}$$

Rough Check: 0.45 hours is a little less than 0.5 or $\frac{1}{2}$ hour. Since $\frac{1}{2}$ hour is 30 minutes, our answer should be a little less than 30 minutes.

Remember that 0.45 hours would be 45 minutes if there were 100 minutes per hour, but since there are only 60 minutes per hour we must remember to take 45/100 of 60.

Example 21

Convert 0.7 pounds to the nearest ounce.

Answer: 11

$$\begin{aligned}0.7 \text{ pounds} &= 0.7 \times 1 \text{ pound} \\ &= 0.7 \times 16 \text{ ounces} \\ &= 11.2 \text{ ounces}\end{aligned}$$

and to the nearest ounce this is 11.

*So 3.7 pounds means
3 pounds 11 ounces, not
3 pounds 7 ounces*

Example 22

Convert 0.7 feet to the nearest tenth of an inch.

Answer: 8.4 (exactly)

$$\begin{aligned}0.7 \text{ feet} &= 0.7 \times 1 \text{ foot} \\ &= 0.7 \times 12 \text{ inches} \\ &= 8.4 \text{ inches}\end{aligned}$$

*So 5.7 feet is 5 ft 8.4 in.
not 5 feet 7 inches*

Notice that while the decimal in Examples 21 and 22 were the same (0.7), the answers were different, because in one case we were taking $\frac{7}{10}$ of 16 (ounces) and in the other case we were taking $\frac{7}{10}$ of 12 (inches). In this sense, metric measurements are easier to master than English measurements.

Example 23

Convert 0.4 centimeters to millimeters.

Answer: 4

We have:

$$\begin{aligned}0.4 \text{ cm} &= 0.4 \times 1 \text{ cm} \\ &= 0.4 \times 10 \text{ mm} \\ &= 4 \text{ mm}\end{aligned}$$

Can you see the advantage of a millimeter being $\frac{1}{10}$ of a centimeter? So in converting from one unit to another in the metric system all we have to do is shift decimal points.

Example 23 is a special case of the rate:

$$\text{number of cm} \times 10 = \text{number of mm}$$

or equivalently:

$$\text{number of mm} \times \frac{1}{10} = \text{number of cm.}$$

Example 24

Convert 24.56 cm to mm.

Answer: 245.6

Just move the decimal point one place
to the right. Namely:

$$\begin{aligned} 24.56 \text{ centimeters} &= 24.56 \times 1 \text{ cm} \\ &= 24.56 \times 10 \text{ mm} \\ &= 245.6 \text{ mm} \end{aligned}$$

Example 25

Convert 24.56 mm to cm.

Answer: 2.456

This time we shift the decimal point one
place to the left. Namely:

$$\begin{aligned} 24.56 \text{ mm} &= 24.56 \times 1 \text{ mm} \\ &= 24.56 \times \frac{1}{10} \text{ cm} \\ &= 24.56 \times 0.1 \text{ cm} \\ &= 2.456 \text{ cm} \end{aligned}$$

Don't memorize if it confuses you. Sheer memory
often causes people to confuse Examples 24 and 25.
Simply use the tables (or look up the rate if it isn't
in the given table) and use the rules of constant rates.

Example 26

Convert 3.547 kilometers to meters?

Answer: 3,547

Simply move the decimal point
three places to the right. But why?
Well look at the rates:

$$\begin{aligned}
 3.547 \text{ km} &= 3.547 \times 1 \text{ kilometer} \\
 &= 3.547 \times 1,000 \text{ meters} \\
 &= 3,547 \text{ meters}
 \end{aligned}$$

Example 27

Convert 3.547 km to cm.

Simply continue what we started in Example 27. Namely:

$$\begin{aligned}
 3.547 \text{ km} &= 3,547 \text{ meters} \\
 &= 3,547 \times 1 \text{ meter} \\
 &= 3,547 \times 100 \text{ cm} \\
 &= 354,700 \text{ cm}
 \end{aligned}$$

Answer: 354,700

In effect, all we had to do was move the decimal point 5 places to the right; but as our solution shows, we don't have to memorize this.

Example 28

Convert 876 mm to km.

We have:

$$\begin{aligned}
 876 \text{ mm} &= 876 \times 1 \text{ mm} \\
 &= 876 \times 0.001 \text{ meters} \\
 &= 0.876 \text{ meters} \\
 &= 0.876 \times 1 \text{ meter} \\
 &= 0.876 \times 0.001 \text{ km} \\
 &= 0.000876 \text{ km}
 \end{aligned}$$

Answer: 0.000876

See how small a millimeter is compared to a kilometer?
 $1 \text{ km} = 1,000 \text{ meters}$
 $= 1,000 \times 1,000 \text{ mm}$
 $= 1,000,000 \text{ mm}$

In other words, an mm is a millionth of a km. Hence to convert mm to kms, we must move the decimal point 6 places to the left.

We could extend this module at length, but the main point has already been made. As a closing series of examples, let's see what you can do with a not-so-common cross-over rate. In the following problems do not get confused by the fact that some of the terms might not be familiar. *Everything you have to know will be told to you.*

Example 29

A hectare is 2.471 acres. How many acres are there in 2,000 hectares?

Answer: 4,942

In terms of "per" we have:

2.471 acres per hectare or

$$\frac{2.471 \text{ acres}}{1 \text{ hectare}}$$

Hence:

$$\frac{2,000 \text{ hectares}}{1} \times \frac{2.471 \text{ acres}}{1 \text{ hectare}} =$$

$$\frac{2,000 \text{ hectares} \times 2.471 \text{ acres}}{1 \times 1 \text{ hectare}} =$$

$$\frac{4,942 \text{ acres}}{1} \text{ or } 4,942 \text{ acres}$$

In other words, we're told in terms of a "recipe" that # of hectares \times 2.471 = # of acres

Once we know this we need no more knowledge about acre and hectare to solve the problem--although we might prefer to have a feeling for what an acre and a hectare are.

Just as a note of interest, an acre is a measure of area in the English system. An acre fits 640 times into a square each of whose sides is a mile long. In other words, an acre is 1/640 of a square mile. A hectare is a measure of area in the metric system. It is the amount of space inside a square each of whose sides has a length of 100 hectameters (100 meters).

Areas will be one of the topics of discussion in the next module.

In this context, 1 hectare = 2.471 acres is the cross-over for converting areas between metric and English units.

Notice that a hectare is larger than an acre. That reminds us to multiply when we convert hectares to acre. Had we wanted to convert acres to hectares we would have to divide by 2.471. This will be illustrated in the next example.

Keeping track of the rates allows us to avoid confusion when Example 29 changes subtly.

Example 30

To the nearest hectare, how many hectares are there in 2,000 acres?

Answer: 809

Start with the relationship that:

Number of hectares X 2.471 = number of acres

and replace the number of acres by 2,000 to
get:

Number of hectares X 2.471 = 2,000

Therefore:

Number of hectares = $2,000 \div 2.471$
= 809.38891....

In other words:

$$\frac{2,000 \text{ acres}}{1} \times \frac{1 \text{ hectare}}{2.471 \text{ acres}} =$$

$$\frac{2,000 \cancel{\text{ acres}} \times 1 \text{ hectare}}{1 \times 2.471 \cancel{\text{ acres}}} =$$

$$\frac{2,000 \text{ hectares}}{2.471} = \frac{2,000,000}{2,471} \text{ hectares}$$

So if you were buying land in Europe (where it is usually sold by the hectare) but you were more comfortable using acres, you'd use the cross-over rate of 2.471 acres per hectare to convert hectare facts into acre facts. This will be emphasized in the Self-Test. But let's not lose sight of our purpose in this module. Our aim was to show you in terms of useful information about measurements and the metric system how we can apply constant rates to the real world in a meaningful way.

Unfortunately not all rates are constant and how we handle such a situation will be the subject of the next module.

In Example 30 we replaced the number of hectares by 2,000.

This is much easier to do if you use a calculator

This is probably the form you'd use if you didn't have a calculator. You'd then get the answer from:

$$2,471 \overline{) 2,000,000}$$